Section 2.3

The Product Rule: The product of two differentiable functions f and g is itself differentiable. Moreover, the derivative of fg is the first function times the derivative of the second, plus the second function times the derivative of the first.

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

1) Find the derivative of the following functions using the Product Rule.

   a) $f(x) = (5x - 4x^3)(2x^4 + 6)$

   b) $g(x) = -5x^3 \cos x$

   c) $h(x) = 3x \sin x + 3 \cos x$

The Quotient Rule: The quotient f/g of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$. Moreover, the derivative of f/g is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}, \qquad g(x) \neq 0$$

2) Find the derivative of the following functions using the Quotient Rule.

   a) $f(x) = \frac{3x^2-7}{2x+5}$

   b) $g(x) = \frac{2x+5}{3-\frac{4}{x}}$

3) Find the equation of the tangent line to $f(x) = \frac{2+x}{1-x}$ at $x = 2$.

Derivatives of Trigonometric Functions:

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$
$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

4) Show that $\frac{d}{dx}[\cot x] = -\csc^2 x$ using the quotient rule.

5) Find the derivative of $f(x) = x^2 \csc x$.

6) If $f(x) = x \sin x$, find the following:

   a) $f'(x)$

   b) $f''(x)$

   c) $f'''(x)$

7) The position function for an object falling from a height of 10 meters on Mars is given by

$$s(t) = -1.85t^2 + 10$$

where $s(t)$ is the height in meters and t is the time in seconds. What is the ratio of Earth's gravitational force to that of Mars?

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #5, 11, 17, 19, 25, 53, 74, 101